Appendix A: Detailed methodology

State-space model

This appendix presents the state-space model used in the paper and discusses the Markov chain Monte Carlo (MCMC) algorithm applied to estimate the state variables.

The notations used in this paper are as follows:

- y_t^{AU} is Australian GDP growth rate in quarter t ($t \in [1,...,T]$).
- y_t^r is the quarterly GSP growth rate for region r ($r \in [1,...,R]$) in quarter t, with $R = 8$.
- $y_t^{r,A}$ is the annual GSP growth rate for region r in quarter t, which is observed in quarter 4 of each year.

Our MF-VAR model is a state-space model with the observed national quarterly GDP and unobserved regional quarterly GSP. The state equation of this state-space model is a VAR model given as:

$$\boldsymbol{y}_t = \Phi_0 + \Phi_1 \boldsymbol{y}_{t-1} + \cdots + \Phi_p \boldsymbol{y}_{t-p}) + \boldsymbol{u}_t, \qquad (A.1)$$

where $\boldsymbol{y}_t = (y_t^{AU}, y_t^1, \ldots, y_t^R)'$ is a $R+1$ vector, and the random term $\boldsymbol{u}_t$ follows $N(0,\Sigma_t)$. The intercept Φ_0 is a $R+1$ vector and the coefficient matrices $\Phi_1, \ldots \Phi_p$ are all $(R+1) \times (R+1)$

To improve modelling performance, this VAR model is expanded to include four additional macroeconomic indicators: the official cash rate, trade-weighted exchange rate, consumer price, and commodity prices, with the latter three indicators entering the model in first difference of logarithm. State final demand for each state are also included. That is, $\boldsymbol{y}_t = (y_t^1, \ldots, y_t^n)'$ with $n = 2R + 5$.

Earlier literatures (Mitchell et al. 2005, Mariano and Murasawa 2010) suggested a linear approximate relationship between the annual GSP growth and quarterly GSP growth:

$$y_t^{r,A} = \tfrac{1}{4}y_t^r + \tfrac{1}{2}y_{t-1}^r + \tfrac{3}{4}y_{t-2}^r + y_{t-3}^r + \tfrac{3}{4}y_{t-4}^r + \tfrac{1}{2}y_{t-5}^r + \tfrac{1}{4}y_{t-6}^r \qquad (A.2)$$

It provides the measurement equations in our state-space model with observed annual growth on the left-hand side and unobserved quarterly growth on the right hand side for each region. Another measurement equation is obtained from the cross-sectional restriction that Australian quarterly GDP growth is the weighted sum of quarterly GSP growth across all the states and territories:

$$y_t^{AU} = \textstyle\sum_{r=1}^{R} w_{r,t} y_t^r + \eta_t, \qquad (A.3)$$

where w_{rt} is set as the region's share of national GSP in the previous year and $\eta_t \sim N(0, \sigma_\eta^2)$.

In most mixed-frequency VAR literature, the covariance matrix Σ_t is assumed to be invariant with time. However, there is evidene of change in volatility in empirical macroeconomic applications. Therefore, we follow a multivariate stochastic volatility specification adopted by Koop et al. (2020). The covariance matrix can be decomposed as follows:

$$\Sigma_t = \boldsymbol{L}'\boldsymbol{D}_t\boldsymbol{L}, \qquad (A.4)$$

where L is a n x n lower triangular matrix with a diagonal of ones and other non-zero elements defined in a vector $\boldsymbol{a} = \left(a_1, \ldots, a_{\frac{(n-1)n}{2}}\right)'$.

The diagonal matrix $\boldsymbol{D}_t = diag[\exp(h_{1,t}), \ldots, \exp(h_{n,t})]$ and the log-volatilities $\boldsymbol{h}_t = (h_{1,t}, \ldots, h_{n,t})]'$ follows a random walk defined as:

$$\boldsymbol{h}_t = \boldsymbol{h}_{t-1} + v_t, v_t \sim \boldsymbol{N}(0,\Sigma_h), \qquad (A.5)$$

where $\Sigma_h = diag[\omega_{h1}^2, \ldots, \omega_{hn}^2]$ is a time-invariant diagonal matrix.v

Priors and posteriors

The goal of our model is to produce posterior and predictive densities for these unobserved quarterly GSP growth and use posterior means as point estimates of these growth rates and densities to produce credible intervals. Bayesian Markov chain Monte Carlo (MCMC) algorithms that combine Bayesian state-space methods with Bayesian VAR methods are used to estimate our model.

The MF-VAR model defined above is obviously overparametrized with $\boldsymbol{n}$ dependent variables and their $\boldsymbol{p}$ lags. In addition, the multivariate stochastic volatility process (A.1) involves more parameters to be estimated ($\boldsymbol{a}$ and $\boldsymbol{h}_t$). To avoid such overparametrisation, we follow Bhattacharya et al. (2015) and use Direchlet-Laplace shrinkage to define priors for all the coefficients in our model.

If we pool all elements of coefficient matrices $(\Phi_0, \ldots \Phi_p)$ together and reshape them into a single vector $\boldsymbol{\phi} = (\phi_1, \ldots \phi_k)'$, where $k = n^2p + n$. The prior for each coefficient is independent and takes the form:

$$\phi_j \sim N\left(0, \psi_j^\phi \vartheta_{j\phi}^2 \tau_\phi^2\right), \qquad (A.6)$$

where the variance involves a local term $\psi_j^\phi \sim \exp\left(\frac{1}{2}\right)$, a global term $\tau_\phi^2 \sim G\left(\kappa\alpha_\phi, \frac{1}{2}\right)$ and an extra term $\vartheta_{j\phi}^2 \sim Dir(\alpha_\phi, \ldots, \alpha_\phi)$. This prior leads to a posterior that contracts to the true value at a rate that is optimal in theory. This prior would shrink the estimate of ϕ_j towards the prior mean of zero relative to maximum likelihood estimate (MLE). This prior involves only one prior hyperparameter $\boldsymbol{a}_\phi$, making the prior elicitation simple. Bhattacharya et al. (2015) recommended setting it to $\frac{1}{2}$ and Koop et al. (2020) approved the selection of prior is reasonably robust.

In addition, we also apply the Dirichlet-Laplace shrinkage to the coefficients in $\boldsymbol{L}$ in equation (A.4). The unknown parameters in log-volatilities is assumed to follow inverse gamma distribution:

$$\omega_{hj}^2 \sim IG(v_{hj}, S_{hj}), \text{ for } j = 1, \ldots, n. \qquad (A.7)$$

The posterior simulation algorithm related to the Dirichlet-Laplace prior is derived in Bhattacharya et al. (2015). Given the draws of state variables, the conditional posterior for the VAR coefficients takes the following form:

$$\phi | \cdot \sim N\left(\hat{\phi}, \boldsymbol{K}_\phi^{-1}\right), \qquad (A.8)$$

where $\boldsymbol{K_\varphi} = \boldsymbol{X}'\boldsymbol{\Sigma}^{-1}\boldsymbol{X} + \boldsymbol{S}_\phi^{-1}$, and $\hat{\phi} = \boldsymbol{K}_\phi^{-1}(\boldsymbol{X}'\boldsymbol{\Sigma}^{-1}\boldsymbol{y})$, with $\boldsymbol{X} = [\boldsymbol{X}_1,\ldots,\boldsymbol{X}_T]$ and $\boldsymbol{X}_t = \boldsymbol{I_n}\otimes[1,\boldsymbol{y'}_{t-1},\ldots,\boldsymbol{y'}_{t-p}]$. The second term in $\boldsymbol{K_\varphi}$ is diagonal, defined as $\boldsymbol{S_\phi} = diag(\psi_1^\phi\vartheta_{1\phi}^2\tau_\phi^2,\ldots,\psi_k^\phi\vartheta_{k\phi}^2\tau_\phi^2)$. The conditional posterior distributions for $\psi_j^\phi, \vartheta_{j\phi}$ and τ_ϕ are:

$$\begin{aligned}
&\psi_j^\phi|\cdot \sim iG((\frac{\upsilon_{j\phi}\tau_\phi)}{|\phi_j|},1);\\
&\tau_\phi|\cdot \sim GIG(k(\alpha_\phi-1),1,2\Sigma_{j=1}^k\frac{|\phi_j|}{\upsilon_{j\phi}});\\
&\upsilon_{j\phi} = \frac{R_{j\phi}}{\Sigma_{j=1}^k R_{j\phi}}
\end{aligned} \qquad (A.9)$$

with $R_{j\phi}\,|.\sim GIG(\alpha_\phi-1, 1, 2|\phi_j|)$, for j=1, ..., k. GIG is the generalised inverse Gaussian distribution and iG is the inverse Gaussian distribution.

Similiarly, the posterior for $\boldsymbol{a}$ is given as

$$\boldsymbol{a}\,|.\sim N(\hat{\boldsymbol{a}}, \boldsymbol{K}_{\boldsymbol{a}}^{-1}) \qquad (A.10)$$

where $\boldsymbol{K_a} = \boldsymbol{E}'\boldsymbol{D}^{-1}\boldsymbol{E} + \boldsymbol{S}_{\boldsymbol{a}}^{-1}$, and $\hat{\boldsymbol{a}} = \boldsymbol{K}_{\boldsymbol{a}}^{-1}(\boldsymbol{E}'\mathbf{D}^{-1}\boldsymbol{\epsilon})$, with $\boldsymbol{D} = \boldsymbol{diag\{D_1,\ldots,D_T\}}$. The detailed definition of matrix $\boldsymbol{E}$ and conditional posteriors for $\boldsymbol{S_a}$ can be found in Koop et al. (2020).

For the stochastic volatility $\boldsymbol{D}_t$, we draw the initial condition $\boldsymbol{h_0}$ following Chan and Eisenstat (2018) and its conditional posterior is:

$$\boldsymbol{h}_0|.\sim N(\hat{\boldsymbol{h}}_0, \boldsymbol{K}_{\boldsymbol{h}_0}^{-1}), \text{ where } \boldsymbol{K}_{\boldsymbol{h}_0} = \boldsymbol{V}_{\boldsymbol{h}}^{-1} + \Sigma_{\boldsymbol{h}}^{-1}, \text{ and } \hat{\boldsymbol{h}}_0 = \boldsymbol{K}_{\boldsymbol{h}_0}^{-1}(\boldsymbol{V}_{\boldsymbol{h}}^{-1}\boldsymbol{a}_{\boldsymbol{h}} + \Sigma_{\boldsymbol{h}}^{-1}\boldsymbol{h}_1). \qquad A.11)$$

The diagonal elements of $\Sigma_{\boldsymbol{h}}$ are conditionally independent and follow:

$$\omega_{h_j}^2|.\sim IG\left(\nu_{hj} + \frac{T}{2},\ S_{hj} + \frac{1}{2}\Sigma_{t=1}^T\left(h_{jt} - h_{j,t-1}\right)^2\right) \text{ for } j = 1,\ldots,n. \qquad (A.12)$$